Exam Quantum Physics 2
January 27, 2026
Start: 15:00h End: 17:00h

Write your name and student ID on each sheet.

INSTRUCTIONS: This is a closed-book and closed-notes exam. You are allowed to bring one A4 page *handwritten* by you, with useful formulas. The exam duration is 2 hours. There is a total of 9 points that you can collect, and you can gain fractions of points by partially answering the questions.

NOTE: If you are not asked to [*Show your work*] then an answer is sufficient. However, you might always earn more points by answering more extensively (but you can also lose points by adding wrong explanations). If you are asked to [*Show your work*], then you should explain your reasoning and the mathematical steps of your derivation in full. Use the official exam paper for *all* your work and ask for more if you need.

USEFUL FORMULAS

Some commutators :

$[x_i, p_j] = i\hbar\delta_{ij}, \quad [L_i, L_j] = i\hbar\epsilon_{ijk}L_k$

Heisenberg EOM, assuming that $A^{(S)}$ is not time dependent :

$$\frac{dA^{(H)}}{dt} = -\frac{i}{\hbar}[A^{(H)}, H^{(H)}], \qquad H^{(H)} = U(t)^\dagger H U(t)$$

Time dependent Schrödinger equation :

$$i\hbar\frac{\partial}{\partial t}|\alpha; t\rangle = H|\alpha; t\rangle$$

Problem 1. (2 points total) Consider rotations and parity transformations in three spatial dimensions. The unitary operator:

$$\mathcal{R}(\hat{\mathbf{n}}, \phi) = e^{-\frac{i}{\hbar}\mathbf{J}\cdot\hat{\mathbf{n}}\phi}, \qquad \mathcal{R}^\dagger = \mathcal{R}^{-1} \tag{1}$$

with $\mathbf{J}$ the angular momentum operator, induces a rotation by an angle ϕ around the direction with unit vector $\hat{\mathbf{n}}$ when acting on a ket-state or a wave function in position space $\psi(\mathbf{x})$. The unitary and Hermitian operator:

$$\mathcal{P}, \quad \mathcal{P}^\dagger = \mathcal{P}^{-1} = \mathcal{P} \tag{2}$$

induces a parity transformation ($\mathbf{x} \to -\mathbf{x}$) when acting on a ket-state or a wave function in position space $\psi(\mathbf{x})$, so that, for example, the parity-transformed wave function is $\psi(-\mathbf{x})$.

a) **[1 points]** Show that two rotations around two different directions of your choice do not commute. [*Show your work*]

b) **[1 points]** Show that parity and rotations do commute. [*Show your work*]

Hint: For both questions a) and b), you can work at the operatorial level, or you can work with a test-function (e.g. $\psi(\mathbf{x})$) or ket upon which the operators act. You can also use infinitesimal forms, where appropriate. Any correct form of physical reasoning is also valued.

Hint: In case it is of use, spherical harmonics transform under parity as: $Y_l^m \to (-1)^l Y_l^m$, as a consequence of the fact that under parity $\mathbf{x} \to -\mathbf{x}$, equivalently, $r \to r$, $\theta \to \pi - \theta$, $\varphi \to \varphi + \pi$.

Problem 2. (4 points total) Consider the valence electron in the $n = 2$ level of the hydrogen atom. Suppose, rather unrealistically, that there is no spin and there are no relativistic effects.

a) **[0.5 points]** What is the degeneracy of the unperturbed energy level?

b) **[1 points]** Add a constant magnetic field with magnitude B in the z-direction, thus inducing a potential $V_B = -\boldsymbol{\mu}_L \cdot \mathbf{B} = -\frac{eB}{2mc} L_z$ ($e < 0$ for an electron). Solve the eigenvalue problem for the system under the effect of B. How are the energy levels in a) modified? Is the degeneracy fully lifted? [*Show your work*]

c) **[2.5 points]** Add to the system in b) a weak uniform electric field with magnitude ε in the z-direction, thus inducing a potential $V_\varepsilon = -e\varepsilon z$ that can be treated as a perturbation. Solve the eigenvalue problem to first order in ε (and $B \neq 0$) and draw the final energy levels. If matrix elements of V_ε happen to vanish, explain why it happens. [*Show your work*]

Hint: You may find it useful to order the states $|l, m\rangle$ as $|0,0\rangle$, $|1,0\rangle$, $|1,1\rangle$, $|1,-1\rangle$.

Hint: m-selection rule:

$$\langle l', m' | T_q^{(k)} | l, m \rangle = 0 \ \text{ unless } m' = m + q \tag{3}$$

where $T_q^{(k)}$ is the component q of a spherical tensor of rank k. For a rank 1 tensor, i.e. a vector, $T_0^{(1)} = \sqrt{\frac{3}{4\pi}} V_z$, $T_{\pm 1}^{(1)} = \mp \sqrt{\frac{3}{4\pi}} \frac{V_x \pm i V_y}{\sqrt{2}}$, with $V_{x,y,z}$ the components of the corresponding cartesian vector.

Hint: In case it is of use, the radial matrix element between $2s$ and $2p$ is $\langle 2s|r|2p\rangle = 3\sqrt{3}a_0$, the angular integral reads:

$$\int d\Omega = \int_0^{2\pi} d\varphi \int_{-1}^{1} d(\cos\theta)$$

$Y_0^0 = \frac{1}{\sqrt{4\pi}}$, $Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$.

Problem 3. (3 points total) Consider the *time-dependent* perturbation

$$V(t) = \begin{cases} V & 0 \leqslant t \leqslant T \\ 0 & \text{otherwise} \end{cases}$$

with V constant and small.

a) **[1 points]** Suppose that the system is in the initial state $|i\rangle$ before the perturbation. For $t \leqslant T$, show that the transition probability from $|i\rangle$ to a final state $|f\rangle$ is:

$$P_{i\to f}(t) = |c_{fi}(t)|^2 = \frac{|V_{fi}|^2}{\hbar^2\omega_{fi}^2}|1 - e^{i\omega_{fi}t}|^2$$

where $V_{fi} = \langle f|V|i\rangle$ and $\omega_{fi} = \frac{E_f - E_i}{\hbar}$.

b) **[1 points]** What happens at times t larger than T? Specifically, what is then the meaning of $c_{fi}(\infty)$? Explain your reasoning.

c) **[1 points]** Consider now the limit $T \to \infty$ in a), i.e. the perturbation is applied at all times $t \geqslant 0$. What is the large time limit of $P_{i\to f}(t)$? For what values of ω_{fi} are transitions possible as $t \to \infty$? [*Show your work*]

Hint:

$$\lim_{\alpha\to\infty} \frac{1}{\pi}\frac{\sin^2 \alpha x}{\alpha x^2} = \delta(x)$$